Produits scalaire et vectoriel

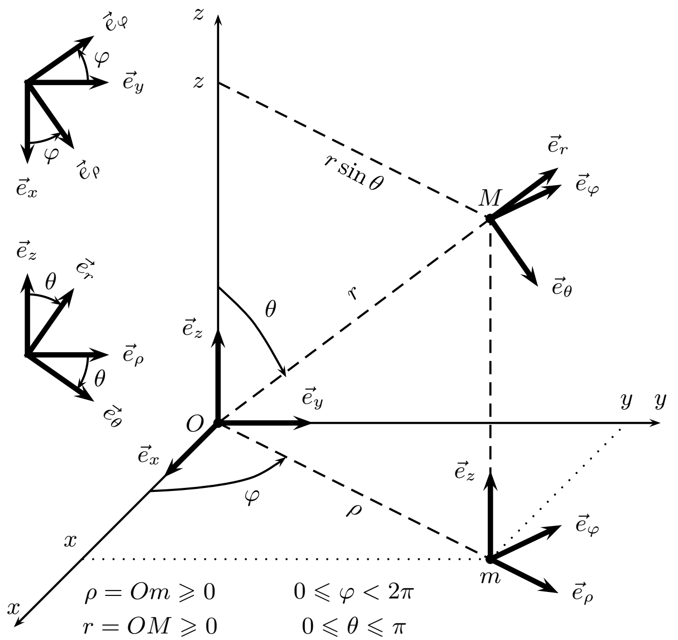
$\vec{a}\cdot\vec{b} = \|\vec{a}\|\times\|\vec{b}\|\times\cos\left(\vec{a},\vec{b}\right)$

$\|\vec{a}\wedge\vec{b}\| = \|\vec{a}\|\times\|\vec{b}\|\times\left|\sin\left(\vec{a},\vec{b}\right)\right|$

$\vec{a}\cdot\left(\vec{b}\wedge\vec{c}\right) = \vec{b}\cdot(\vec{c}\wedge\vec{a}) = \vec{c}\cdot\left(\vec{a}\wedge\vec{b}\right) = \det\left(\vec{a},\vec{b},\vec{c}\right) = \pm\mathrm{vol}\left(\vec{a},\vec{b},\vec{c}\right)$

$\vec{a}\wedge(\vec{b}\wedge\vec{c}) = \vec{b}\times(\vec{a}\cdot\vec{c}) - \vec{c}\times(\vec{a}\cdot\vec{b})$

Systèmes de coordonnées orthogonaux



$\rho = Om \geqslant 0 \qquad 0 \leqslant \varphi < 2\pi$

$r = OM \geqslant 0 \qquad 0 \leqslant \theta \leqslant \pi$

$\vec{e}_\rho = \vec{e}_x\cos\varphi + \vec{e}_y\sin\varphi,\ \vec{e}_\varphi = -\vec{e}_x\sin\varphi + \vec{e}_y\cos\varphi$

$\vec{e}_r = \vec{e}_z\cos\theta + \vec{e}_\rho\sin\theta,\ \vec{e}_\theta = -\vec{e}_z\sin\theta + \vec{e}_\rho\cos\theta$

$x = \rho\cos\varphi,\ y = \rho\sin\varphi,\ z = r\cos\theta,\ \rho = r\sin\theta$

$\mathrm{d}\vec{r} = \mathrm{d}x\vec{e}_x + \mathrm{d}y\vec{e}_y + \mathrm{d}z\vec{e}_z\ ;\ \mathrm{d}\tau = \mathrm{d}x\times\mathrm{d}y\times\mathrm{d}z$

$\mathrm{d}\vec{r} = \mathrm{d}\rho\vec{e}_\rho + \rho\mathrm{d}\varphi\vec{e}_\varphi + \mathrm{d}z\vec{e}_z\ ;\ \mathrm{d}\tau = \rho\,\mathrm{d}\rho\times\mathrm{d}\varphi\times\mathrm{d}z$

$\mathrm{d}\vec{r} = \mathrm{d}r\vec{e}_r + r\mathrm{d}\theta\vec{e}_\theta + r\sin\theta\mathrm{d}\varphi\vec{e}_\varphi\ ;\ \mathrm{d}\tau = r^2\,\mathrm{d}r\times\sin\theta\,\mathrm{d}\theta\times\mathrm{d}\varphi$

Opérateurs différentiels

$\mathrm{d}F = \overrightarrow{\mathrm{grad}}\,F\cdot\mathrm{d}\vec{r}$; $\overrightarrow{\mathrm{grad}}\,F = \vec{\nabla}F = \dfrac{\partial F}{\partial x}\vec{e}_x + \dfrac{\partial F}{\partial y}\vec{e}_y + \dfrac{\partial F}{\partial z}\vec{e}_z$

$\displaystyle\oint_{\mathcal{S}}\vec{V}\cdot\mathrm{d}\vec{S} = \int_{\mathcal{V}}\mathrm{div}\,\vec{V}\mathrm{d}\tau$ (Ostrogradski ; $\mathcal{S}$ est fermée et délimite le volume intérieur $\mathcal{V}$) ; $\mathrm{div}\,\vec{V} = \vec{\nabla}\cdot\vec{V} = \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} + \dfrac{\partial V_z}{\partial z}$

$\displaystyle\oint_\Gamma\vec{V}\cdot\mathrm{d}\vec{r} = \int_\Sigma\overrightarrow{\mathrm{rot}}\,\vec{V}\mathrm{d}\vec{S}$ (Stokes ; Γ est fermée et constitue le bord orienté de Σ) ; $\overrightarrow{\mathrm{rot}}\,\vec{V} = \vec{\nabla}\wedge\vec{V} = \left[\dfrac{\partial V_z}{\partial y} - \dfrac{\partial V_y}{\partial z}\right]\vec{e}_x + \ldots$

$\Delta F = \mathrm{div}\,\overrightarrow{\mathrm{grad}}\,F\ ;\ \Delta F = \nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}$

$\overrightarrow{\mathrm{rot}}\,\overrightarrow{\mathrm{rot}}\,\vec{V} = \overrightarrow{\mathrm{grad}}\,\mathrm{div}\,\vec{V} - \Delta\vec{V}\ ;\ \Delta\vec{V} = \nabla^2\vec{V} = \Delta V_x\vec{e}_x + \ldots$

$\mathrm{d}\vec{V} = \left(\mathrm{d}\vec{r}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V}\ ;\ \left(\vec{a}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V} = \left(\vec{a}\cdot\vec{\nabla}\right)\vec{V} = a_x\frac{\partial\vec{V}}{\partial x} + \ldots$

Coordonnées cylindro-polaires

$$\overrightarrow{\mathrm{grad}}\,F = \frac{\partial F}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial F}{\partial\varphi}\vec{e}_\varphi + \frac{\partial F}{\partial z}\vec{e}_z$$

$$\mathrm{div}\,\vec{V} = \frac{1}{\rho}\left\{\frac{\partial}{\partial\rho}\left(\rho V_\rho\right) + \frac{\partial V_\varphi}{\partial\varphi}\right\} + \frac{\partial V_z}{\partial z}$$

$$\overrightarrow{\mathrm{rot}}\,\vec{V} = \left\{\frac{1}{\rho}\frac{\partial V_z}{\partial\varphi} - \frac{\partial V_\varphi}{\partial z}\right\}\vec{e}_\rho + \left\{\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial\rho}\right\}\vec{e}_\varphi + \ldots$$

$$\ldots + \frac{1}{\rho}\left\{\frac{\partial}{\partial\rho}\left(\rho V_\varphi\right) - \frac{\partial V_\rho}{\partial\varphi}\right\}\vec{e}_z$$

$$\Delta F = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial F}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 F}{\partial\varphi^2} + \frac{\partial^2 F}{\partial z^2}$$

$$\Delta F(\rho) = 0 \Rightarrow F(\rho) = A\ln\rho \Rightarrow \vec{V} = \overrightarrow{\mathrm{grad}}\,F = A\vec{e}_\rho/\rho$$

Coordonnées sphériques

$$\overrightarrow{\mathrm{grad}}\,F = \frac{\partial F}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial F}{\partial\theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial F}{\partial\varphi}\vec{e}_\varphi$$

$$\mathrm{div}\,\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\left\{\frac{\partial}{\partial\theta}\left(\sin\theta V_\theta\right) + \frac{\partial V_\varphi}{\partial\varphi}\right\}$$

$$\overrightarrow{\mathrm{rot}}\,\vec{V} = \frac{1}{r\sin\theta}\left\{\frac{\partial}{\partial\theta}\left(\sin\theta V_\varphi\right) - \frac{\partial V_\theta}{\partial\varphi}\right\}\vec{e}_r + \ldots$$

$$\ldots + \frac{1}{r}\left\{\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\varphi} - \frac{\partial}{\partial r}\left(rV_\varphi\right)\right\}\vec{e}_\theta + \frac{1}{r}\left\{\frac{\partial}{\partial r}\left(rV_\theta\right) - \frac{\partial V_r}{\partial\theta}\right\}\vec{e}_\varphi$$

$$\Delta F = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 F}{\partial\varphi^2}$$

$$\Delta F(r) = 0 \Rightarrow F(r) = -A/r \Rightarrow \vec{V} = \overrightarrow{\mathrm{grad}}\,F = A\vec{e}_r/r^2$$

Propriétés générales

$\overrightarrow{\mathrm{grad}}\left(\overrightarrow{\mathrm{Cte}}\cdot\vec{r}\right) = \overrightarrow{\mathrm{Cte}}\ ;\ \overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{Cte}}\wedge\vec{r}\right) = 2\times\overrightarrow{\mathrm{Cte}}\ ;\ \mathrm{div}\,\vec{r} = 3$

$\overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{grad}}\,F\right) = 0\ ;\ \overrightarrow{\mathrm{rot}}\,\vec{y} = 0 \Rightarrow \exists x\,/\,\vec{y} = \overrightarrow{\mathrm{grad}}\,x$

$\mathrm{div}\left(\overrightarrow{\mathrm{rot}}\,\vec{V}\right) = 0\ ;\ \mathrm{div}\,\vec{y} = 0 \Rightarrow \exists\vec{x}\,/\,\vec{y} = \overrightarrow{\mathrm{rot}}\,\vec{x}$

$\overrightarrow{\mathrm{grad}}\,(F\,G) = F\,\overrightarrow{\mathrm{grad}}\,G + G\,\overrightarrow{\mathrm{grad}}\,F$

$\mathrm{div}\left(F\,\vec{V}\right) = F\,\mathrm{div}\,\vec{V} + \vec{V}\cdot\overrightarrow{\mathrm{grad}}\,F$

$\mathrm{div}\left(\vec{U}\wedge\vec{V}\right) = \vec{V}\cdot\overrightarrow{\mathrm{rot}}\,\vec{U} - \vec{U}\cdot\overrightarrow{\mathrm{rot}}\,\vec{V}$

$\overrightarrow{\mathrm{rot}}\left(F\,\vec{V}\right) = F\,\overrightarrow{\mathrm{rot}}\,\vec{V} + \overrightarrow{\mathrm{grad}}\,F\wedge\vec{V}$

$\overrightarrow{\mathrm{grad}}\left(\vec{U}\cdot\vec{V}\right) = \vec{U}\wedge\overrightarrow{\mathrm{rot}}\,\vec{V} + \vec{V}\wedge\overrightarrow{\mathrm{rot}}\,\vec{U} + \left(\vec{V}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{U} + \left(\vec{U}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V}$

$\overrightarrow{\mathrm{rot}}\left(\vec{U}\wedge\vec{V}\right) = \left(\mathrm{div}\,\vec{V}\right)\vec{U} - \left(\mathrm{div}\,\vec{U}\right)\vec{V} - \left(\vec{U}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{V} + \ldots$

$\ldots + \left(\vec{V}\cdot\overrightarrow{\mathrm{grad}}\right)\vec{U}$

Théorèmes intégraux

Γ est fermée et constitue le bord orienté de Σ.

Stokes : $\displaystyle\oint_\Gamma\vec{V}\cdot\mathrm{d}\vec{r} = \int_\Sigma\overrightarrow{\mathrm{rot}}\,\vec{V}\mathrm{d}\vec{S}$

Kelvin : $\displaystyle\oint_\Gamma F\,\mathrm{d}\vec{r} = \int_\Sigma\mathrm{d}\vec{S}\wedge\overrightarrow{\mathrm{grad}}\,F$

$\mathcal{S}$ est fermée et délimite le volume intérieur $\mathcal{V}$.

Ostrogradski : $\displaystyle\oint_{\mathcal{S}}\vec{V}\cdot\mathrm{d}\vec{S} = \int_{\mathcal{V}}\mathrm{div}\,\vec{V}\mathrm{d}\tau$

Gradient : $\displaystyle\oint_{\mathcal{S}}F\,\mathrm{d}\vec{S} = \int_{\mathcal{V}}\overrightarrow{\mathrm{grad}}\,F\mathrm{d}\tau$

Primitives usuelles

Fonction	Primitive
$(x-a)^{n,\ n\neq-1}$	$\dfrac{1}{n+1}(x-a)^{n+1}$
$\dfrac{1}{x-a}$	$\ln\lvert x-a\rvert$
$\exp(ax)$	$\dfrac{1}{a}\exp(ax)$
$\ln x$	$x\ln x - x$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$-\ln\lvert\cos x\rvert$
$\dfrac{1}{\tan x}$	$\ln\lvert\sin x\rvert$
$1/\cos^2 x$	$\tan x$
$1/\sin^2 x$	$-\dfrac{1}{\tan x}$
$1/\cos x$	$\ln\left\lvert\tan\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right\rvert$
$1/\sin x$	$\ln\left\lvert\tan\dfrac{x}{2}\right\rvert$
$\mathrm{ch}\,x$	$\mathrm{sh}\,x$
$\mathrm{sh}\,x$	$\mathrm{ch}\,x$
$1/\mathrm{ch}^2 x$	$\mathrm{th}\,x$
$1/\mathrm{sh}^2 x$	$-\dfrac{1}{\mathrm{th}\,x}$
$1/\mathrm{ch}\,x$	$2\arctan(\exp(x))$
$1/\mathrm{sh}\,x$	$\ln\left\lvert\mathrm{th}\,\dfrac{x}{2}\right\rvert$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\arctan\dfrac{x}{a}$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\mathrm{argth}\,\dfrac{x}{a}$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\ln\left(\dfrac{x}{a}+\sqrt{\dfrac{x^2}{a^2}+1}\right) = \mathrm{argsh}\,\dfrac{x}{a}$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\arcsin\dfrac{x}{a}$
$\left(1\pm x^2\right)^{-3/2}$	$\dfrac{x}{\sqrt{1\pm x^2}}$

Fonctions de Bessel

Équation de Bessel : $x^2y''+xy'+(x^2-\nu^2)y=0$

Solution générale : $y(x)=\alpha J_\nu(x)+\beta Y_\nu(x)$

$J_\nu(x)\sim_{x\to0}\dfrac{x^\nu}{2^\nu\nu!}$; $Y_\nu(x)\sim_{x\to0}-\dfrac{2^\nu(\nu-1)!}{\pi x^\nu}$

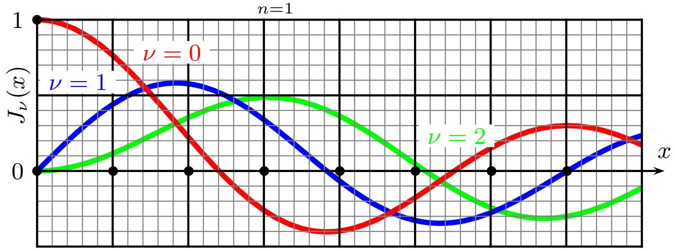
$J_{\nu+1}(x)=\dfrac{2\nu}{x}J_\nu(x)-J_{\nu-1}(x)$, $Y_{\nu+1}(x)=\dfrac{2\nu}{x}Y_\nu(x)-Y_{\nu-1}(x)$

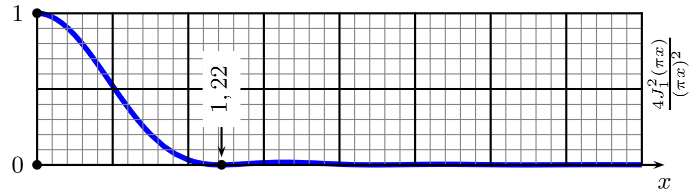
$\dfrac{\mathrm{d}J_\nu}{\mathrm{d}x}=\dfrac{J_{\nu+1}(x)-J_{\nu-1}(x)}{2}$, $\dfrac{\mathrm{d}Y_\nu}{\mathrm{d}x}=\dfrac{Y_{\nu+1}(x)-Y_{\nu-1}(x)}{2}$

$$J_\nu(x)=\frac{1}{\pi}\int_0^\pi\cos\left(\nu\theta-x\sin\theta\right)\mathrm{d}\theta$$

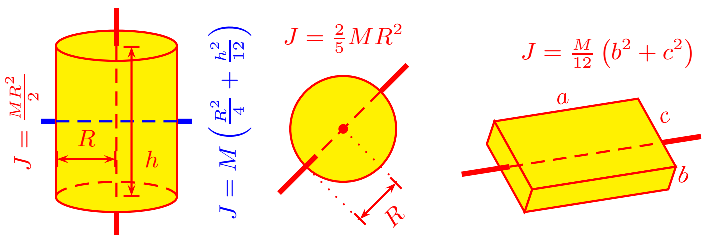
$$\sin\left(x\sin\theta\right)=2\sum_{n=1}^{\infty}J_{2n-1}(x)\sin\left([2n-1]\theta\right)$$

$$\cos\left(x\sin\theta\right)=J_0(x)+2\sum_{n=1}^{\infty}J_{2n}(x)\cos\left(2n\theta\right)$$

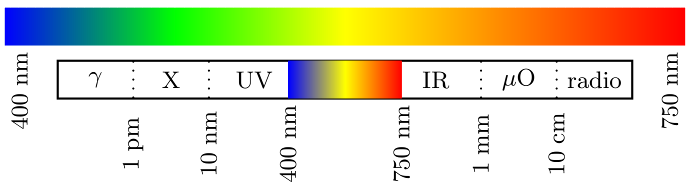




Moments d'inertie de solides pleins

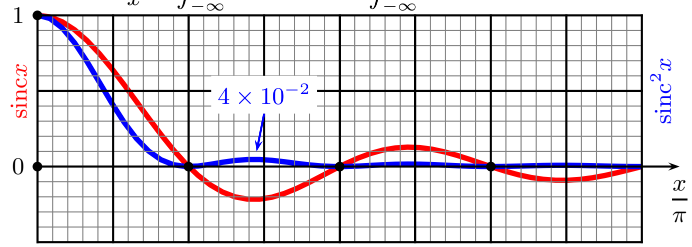


Spectre électromagnétique

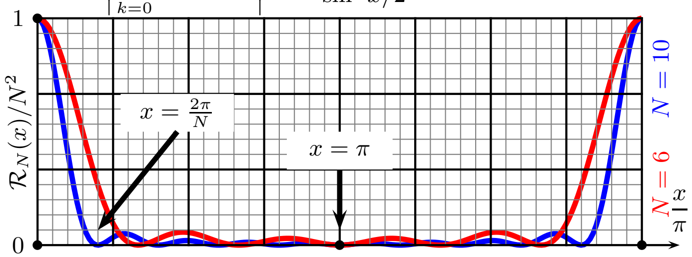


Fonctions de l'Optique

$$\mathrm{sinc}(x)=\frac{\sin x}{x},\ \int_{-\infty}^{\infty}\mathrm{sinc}(x)\,\mathrm{d}x=\int_{-\infty}^{\infty}\mathrm{sinc}^2(x)\,\mathrm{d}x=\pi$$

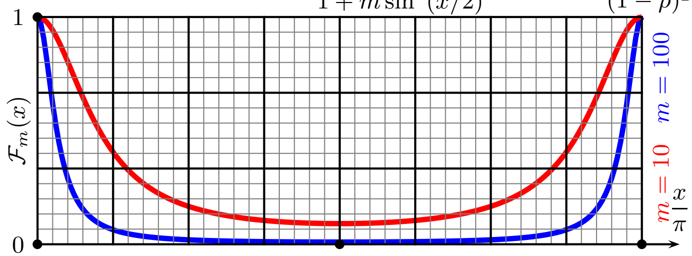


$$\mathcal{R}_N(x)=\left|\sum_{k=0}^{N-1}\exp\left(ikx\right)\right|^2=\frac{\sin^2Nx/2}{\sin^2x/2}$$



$$\left|\sum_{k=0}^{\infty}\rho^k\exp\left(ikx\right)\right|^2=\frac{1}{(1-\rho)^2}\mathcal{F}_m(x)\text{ si }\rho<1$$

$$\mathcal{F}_m(x)=\frac{1}{1+m\sin^2(x/2)}\text{ avec }m=\frac{4\rho}{(1-\rho)^2}$$



Classification périodique des éléments

non-métaux

semi-conducteurs

métaux

H																		He
Li	Be													B	C	N	O	F	Ne
Na	Mg													Al	Si	P	S	Cl	Ar
K	Ca		Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr		Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	*	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	+	Lr	Rf	Ha	Sg	Ns	Hs	Mt									

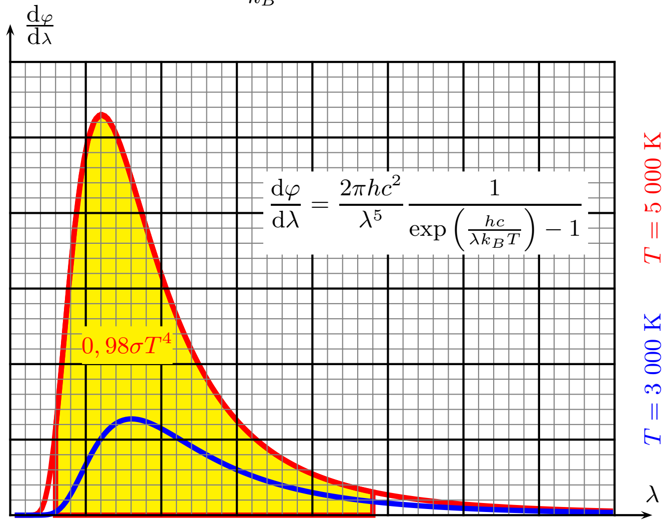
*	La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	lanthanides
+	Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	actinides

Rayonnement thermique

$$\frac{\mathrm{d}u}{\mathrm{d}\nu}=\frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{k_BT}\right)-1}\ ;\ \frac{\mathrm{d}\varphi}{\mathrm{d}\nu}=\frac{c}{4}\frac{\mathrm{d}u}{\mathrm{d}\nu}$$

$$\int_0^\infty\frac{x^3\mathrm{d}x}{\exp(x)-1}=\frac{\pi^4}{15},\ \sigma=\frac{2\pi^5k_B^4}{15c^2h^3}$$

$$\lambda_{\max}T=C_W=0,201\frac{hc}{k_B}=2,90\times10^{-3}\ \mathrm{m\cdot K}$$



Constantes fondamentales

- $c=3,00\times10^{8}$ m · s^{-1}
- $e=1,60\times10^{-19}$ C
- $\epsilon_0=8,85\times10^{-12}$ F·m^{-1}
- $\mathcal{F}=96\,500$ C · mol^{-1}
- $R=8,31$ J · K^{-1} · mol^{-1}
- $h=6,63\times10^{-34}$ J · s
- $k_B=1,38\times10^{-23}$ J·K^{-1}
- $m_e=9,11\times10^{-31}$ kg
- $m_p\simeq m_n\simeq1,67\times10^{-27}$ kg
- $\mu_0=4\times\pi\times10^{-7}$ H · m^{-1}
- $\mathcal{N}_A=6,02\times10^{23}$ mol^{-1}
- $\mathcal{G}=6,67\times10^{-11}$ m^3 · kg^{-1} · s^{-2}
- $\sigma=5,67\times10^{-8}$ W · m^{-2} · K^{-4}
- $T_T=273,16$ K

Données astronomiques

- $M_\odot=1,99\times10^{30}$ kg
- 1 UA $=1,50\times10^{11}$ m
- 1 pc $=3,09\times10^{16}$ m
- 1 an $=365,25$ j (solaire)
- $R_\odot=6,96\times10^{8}$ m
- 1 AL $=9,46\times10^{15}$ m
- 1 j (solaire) $=86\,400$ s
- 1 j (sidéral) $=86\,164$ s

Terre

- $M=5,98\times10^{24}$ kg
- $R=6,38\times10^{6}$ m
- $d_\odot=1$ UA
- $e=0,017$
- $T=1$ an

Lune

- $M=7,35\times10^{22}$ kg
- $R=1,74\times10^{6}$ m
- $d_{\mathrm{Terre}}=3,84\times10^{8}$ m
- $e=0,055$
- $T=27,3$ j (solaire)